Q:→ $(D^4 + D^2 + 1)\, y = ax^2 + b e^{-x} \sin 2x$

Soln:→ A.E. is

$$m^4 + m^2 + 1 = 0$$

$$m^4 + 2m^2 + 1 - m^2 = 0$$

$$(m^2+1)^2 - m^2 = 0$$

$$(m^2+1+m)(m^2+1-m) = 0$$

$$(m^2+m+1)(m^2-m+1) = 0$$

$$m = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}\,i}{2} = \frac{-1}{2} \pm \frac{\sqrt{3}}{2} i$$

$$m = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{3}\,i}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

$$\boxed{\begin{aligned} C.F. = {} & e^{-\frac{1}{2}x}\left[c_1 \cos\left(\tfrac{\sqrt{3}}{2}x\right) + c_2 \sin\left(\tfrac{\sqrt{3}}{2}x\right)\right] \\ & + e^{\frac{1}{2}x}\left[c_3 \cos\left(\tfrac{\sqrt{3}}{2}x\right) + c_4 \sin\left(\tfrac{\sqrt{3}}{2}x\right)\right] \end{aligned}}$$

$$P.I. = \frac{1}{D^4+D^2+1}\left[ax^2 + b e^{-x} \sin 2x\right]$$

$$P.I. = a \frac{1}{D^4+D^2+1} x^2 + b \frac{1}{D^4+D^2+1} e^{-x} \sin 2x$$

let $P.I = a I_1 + b I_2$

Where $I_1 = \frac{1}{D^4+D^2+1} x^2$; $I_2 = \frac{1}{D^4+D^2+1} e^{-x} \sin 2x$

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$$I_1 = \frac{1}{D^4+D^2+1} x^2$$

$$= \left[1+(D^4+D^2)\right]^{-1} x^2$$

$$= \left[1-(D^4+D^2)+(D^4+D^2)^2 - \cdots\right] x^2$$

$$= \left[x^2 - [0+2x] + 0 + \cdots\right]$$

$$I_1 = x^2 - 2x$$

$$I_2 = \frac{1}{D^4+D^2+1} e^{-x} \sin 2x$$

$D \to D-1$

$$= e^{-x} \frac{1}{(D-1)^4+(D-1)^2+1} \sin 2x$$

$$= e^{-x} \frac{1}{(D^4-4D^3+6D^2-4D+1)+(D^2-2D+1)+1} \sin 2x$$

$$= e^{-x} \frac{1}{D^4-4D^3+7D^2-6D+3} \sin 2x$$

$D^2 = -4$

$$= e^{-x} \frac{1}{(-4)^2-4(-4)D+7(-4)-6D+3} \sin 2x$$

$$= e^{-x} \frac{1}{16+16D-28-6D+3} \sin 2x$$

$$= e^{-x} \frac{1}{10D-9} \sin 2x$$

$$= e^{-x} \frac{10D+9}{(10D+9)(10D-9)} \sin 2x = e^{-x} \frac{(10D+9)}{100D^2-81} \sin 2x$$

$D^2 \to -4$

$$= e^{-x} \frac{(10D+9)\sin 2x}{-400-81} = \frac{e^{-x}}{-481}\left[20\cos 2x + 9\sin 2x\right]$$

Hence soln

$$\boxed{y = C.F. + P.I.}$$

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